

# First Course on POWER SYSTEMS

MNPERE

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Course Objective #3

# Review Elements of Basic Electric Circuit Theory Essential in Learning Power Systems

CONSTANTIN APOSTOIA, PhD

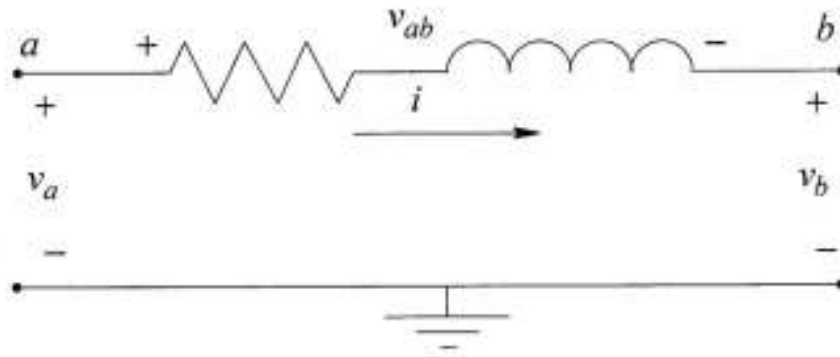
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# Outline

- General use of phasors for analysis of electric circuits in sinusoidal steady-state
- Instantaneous, average, and reactive powers
- Power factor
- Analysis of 3-phase circuits
- Power flow in ac circuits
- Per-unit quantities

# Symbols and Conventions

- Use of MKS (SI) units, and IEEE standard letters and symbols
- Current direction, voltage polarities
- Passive sign convention



Convention for voltages and currents

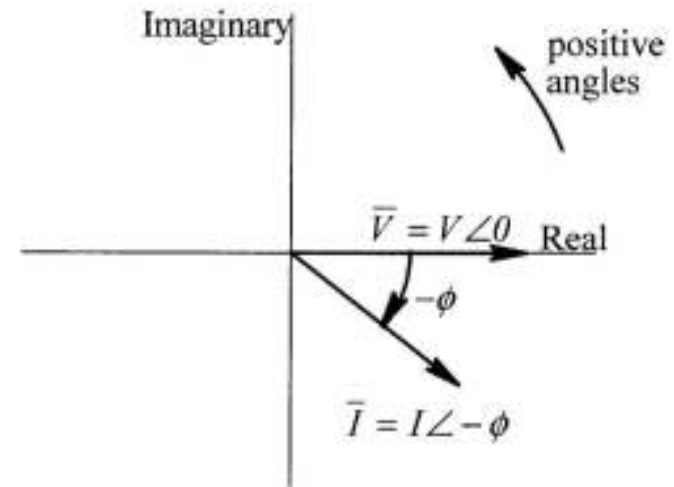
❖ Note:  $v_{ab} = v_a - v_b$

# Phasor Representation in Sinusoidal Steady State

- Voltages and currents applied for a long time, with  $f = \omega/(2\pi)$
- Simplified circuit calculations by means of phasor-domain analysis
- Use of *rms* values for  $V$  and  $I$  as magnitudes of complex variables

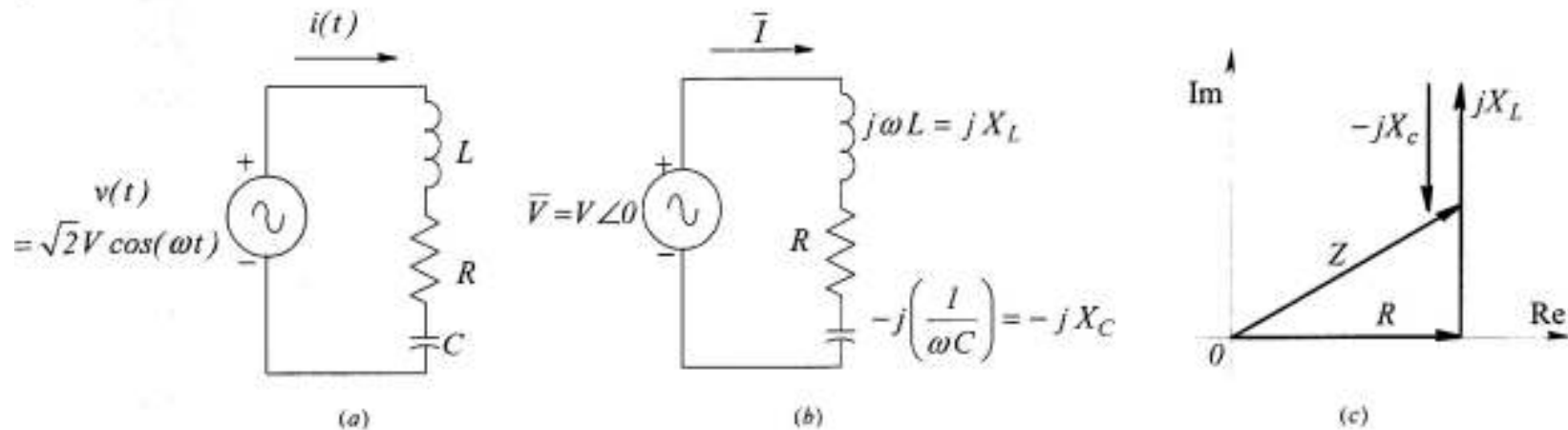
$$v(t) = \sqrt{2}V\cos(\omega t)$$

$$i(t) = \sqrt{2}I\cos(\omega t - \Phi)$$



Phasor diagram

# Phasor-Domain Analysis



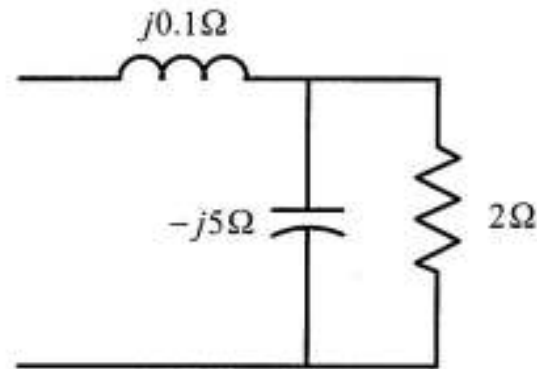
A circuit in sinusoidal steady-state  
(a) in time-domain, and (b) in phasor-domain; (c) impedance triangle

❖ Discussion question:

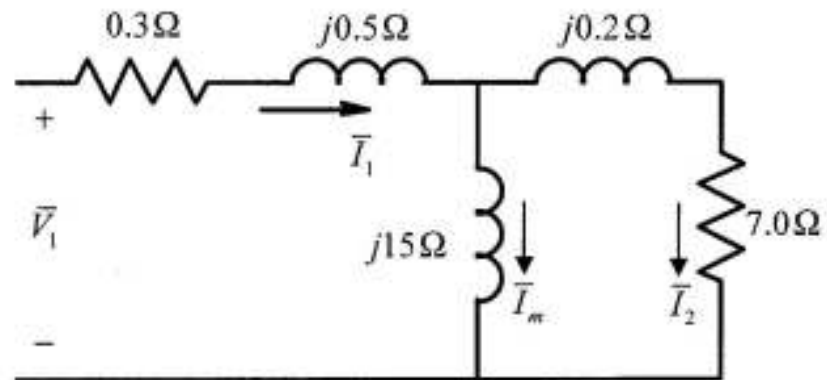
Do we want to include, eventually, the minus sign for  $X_C = -1/(\omega C)$  ?

# Examples of Impedance Calculation

- calculate the equivalent impedance



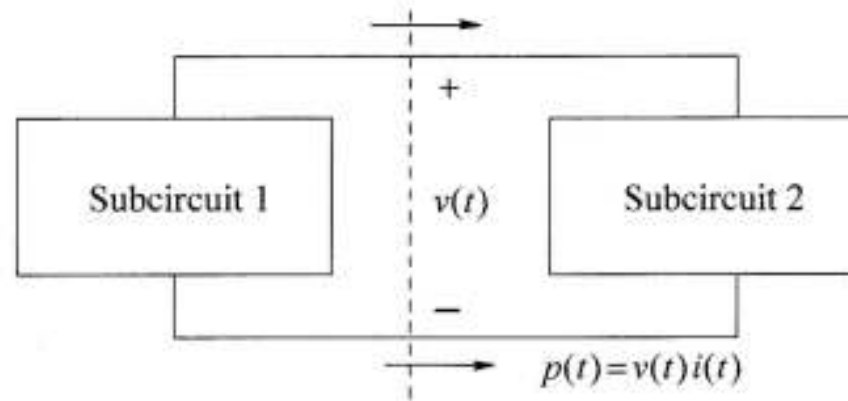
- Given  $V_1$  and frequency  $f$ , calculate  $I_1$



❖Note: Impedance  $Z$ , and admittance  $Y = 1/Z$ , are complex quantities and not phasors.

# Power Flow

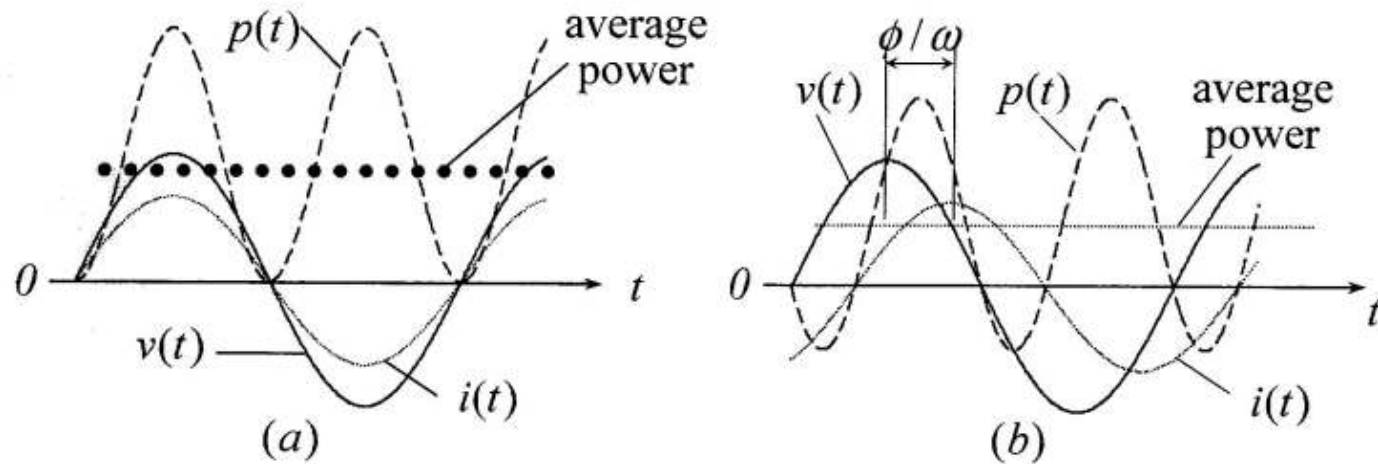
- each subcircuit may consist of passive elements and active voltage and/or current sources
- arbitrary  $v$ , and  $i$
- a negative value of  $p(t)$  reverses the assumed roles of both subcircuits.



A generic circuit divided into two sub-circuits



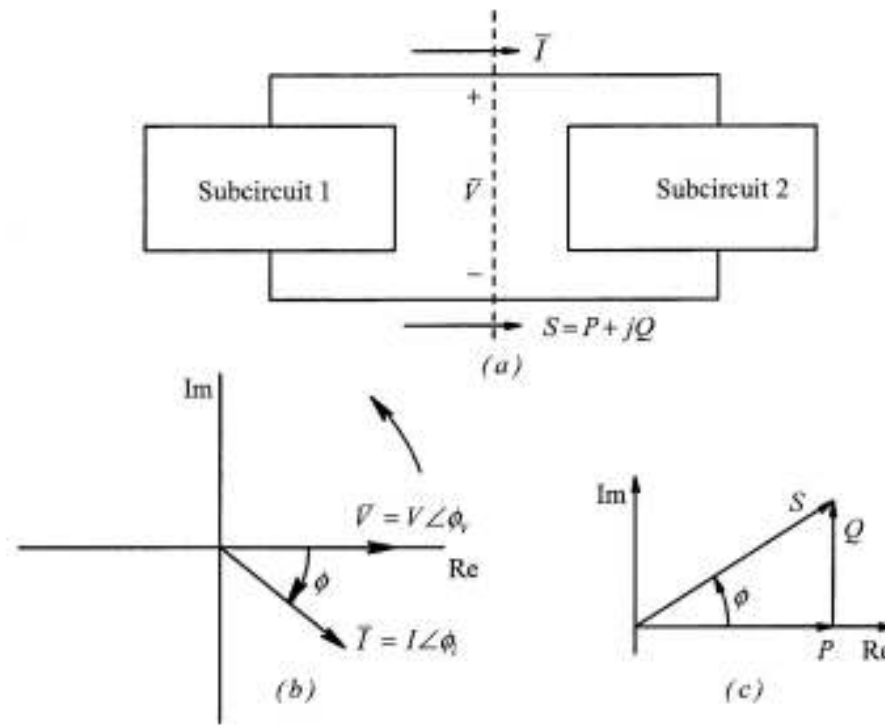
# Instantaneous and Average Power



Instantaneous power with sinusoidal currents and voltages

- (a)  $v$  and  $i$  are in phase,  $p(t) \geq 0$ , always, and average is  $P=VI$
- (b)  $i$  lags behind  $v$  by  $\Phi(t)$ , when  $p(t) < 0$  power flows in the opposite direction, and average is  $P=VI\cos\Phi$

# Complex, Average (Real), Reactive, and Apparent Power



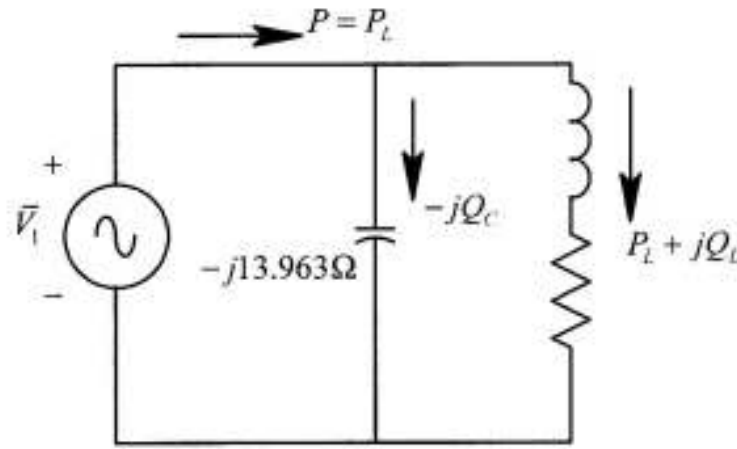
(a) Circuit in phasor-domain; (b) phasor diagram;  
(c) power triangle

❖Note: Power balance for the total average and reactive supplied powers

# Example

## Power Factor Correction

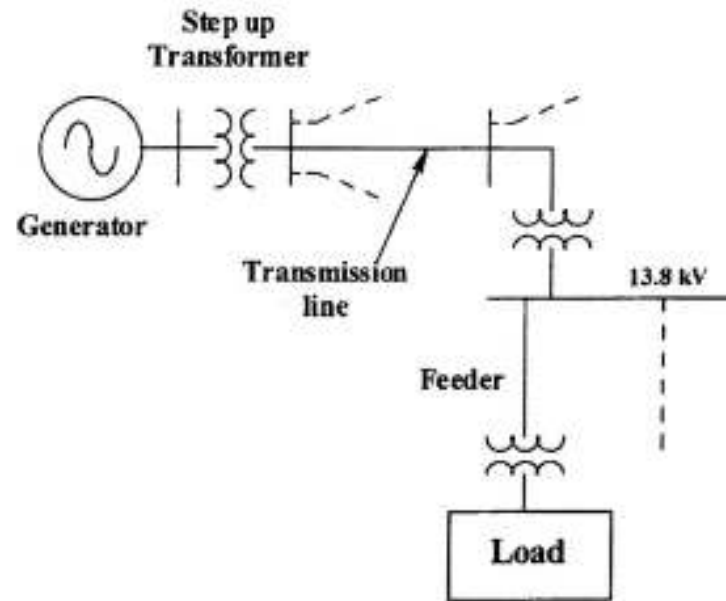
- Calculate the reactance in parallel with the load that will result in the overall pf = 1 (or close) as seen from the voltage source



Power factor correction

- ❖ Note: derive a practical formula of the required capacitance for the power factor improvement ( $\cos\Phi_{\text{old}} < \cos\Phi_{\text{new}}$ )

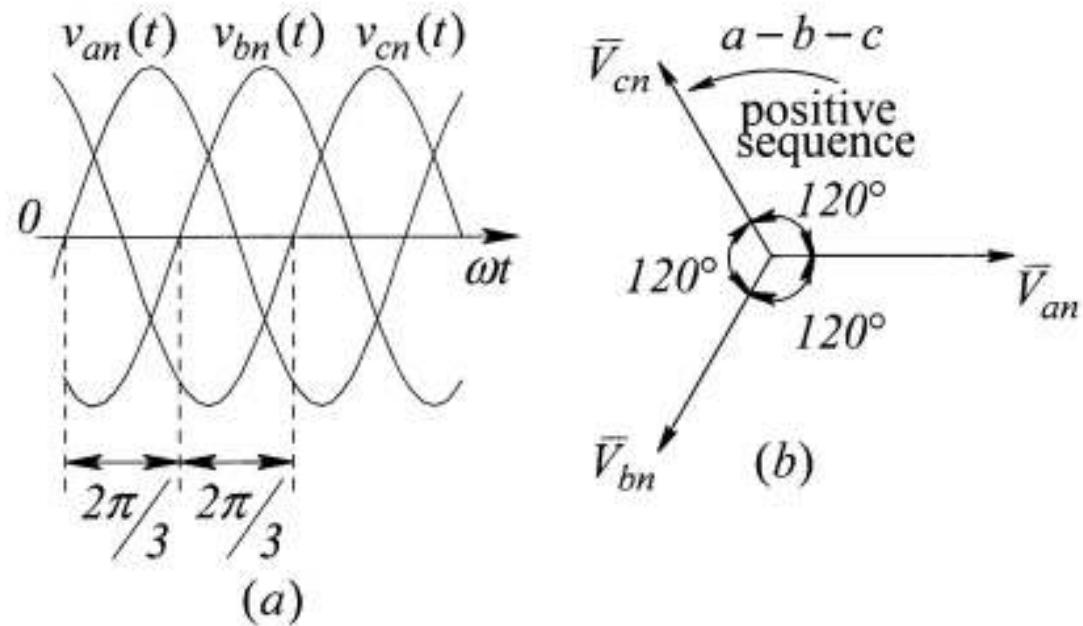
# Three-Phase Circuits



One-line diagram of a three-phase transmission and distribution system

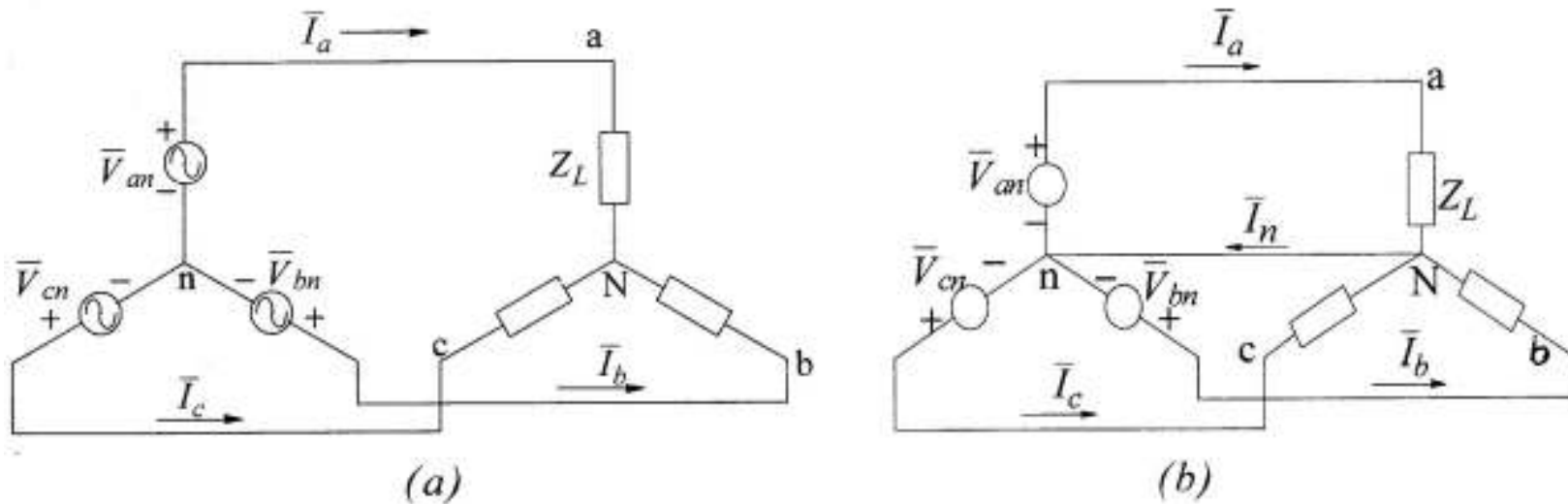
- Most of electric generators are ac 3-phase generators (usually  $< 25$  kV)

# Three-Phase Voltages



Three-phase voltages (a) in time domain and (b) phasor domain

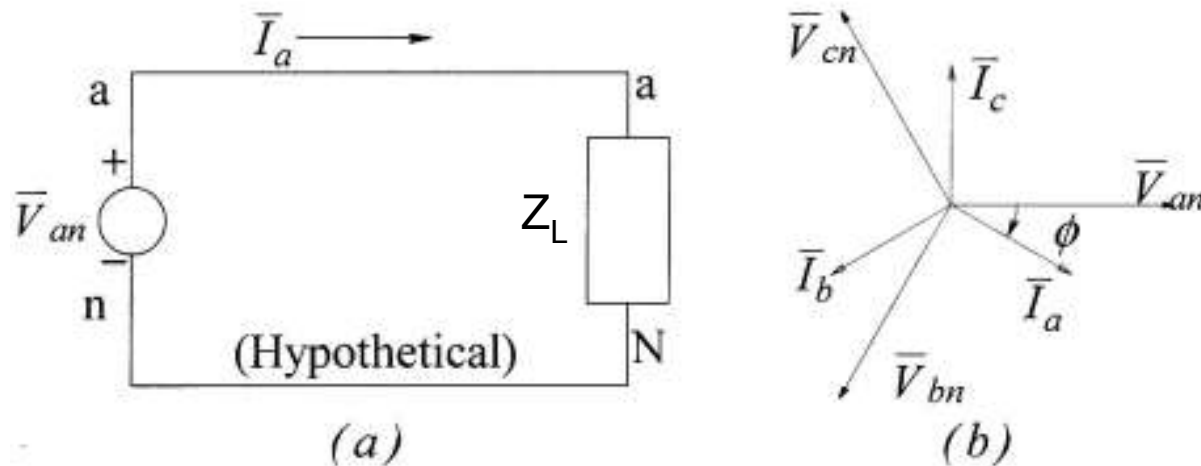
# Balanced Three-Phase Circuit Analysis



A balanced wye-connected, three-phase circuit

- Use of the 4<sup>th</sup> wire ( $V_{nN} = 0$ ) leads to per-phase circuit analysis

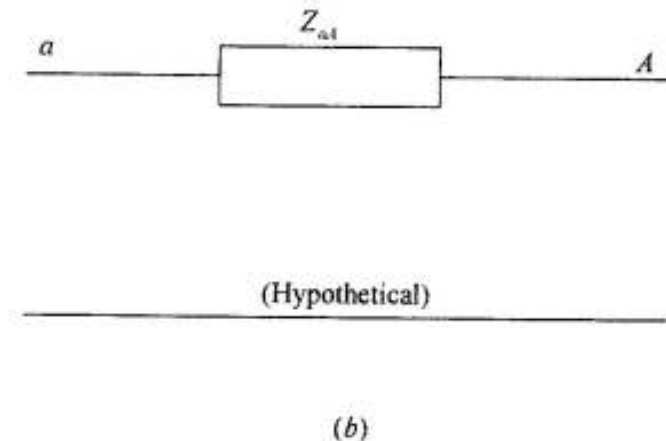
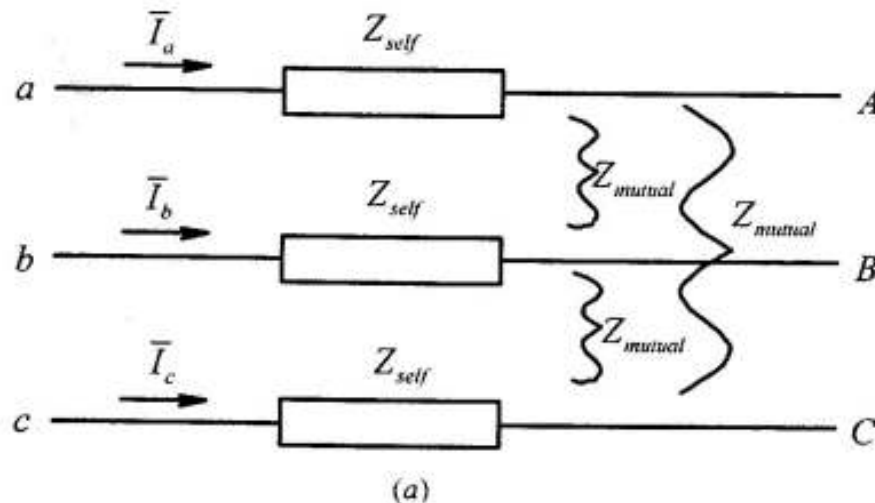
# Per-Phase Analysis



Per-phase circuit and the corresponding phasor diagram

- The total power factor is the same as its per-phase value
- The total powers  $P_T = 3P_{ph}$ ;  $Q_T = 3Q_{ph}$

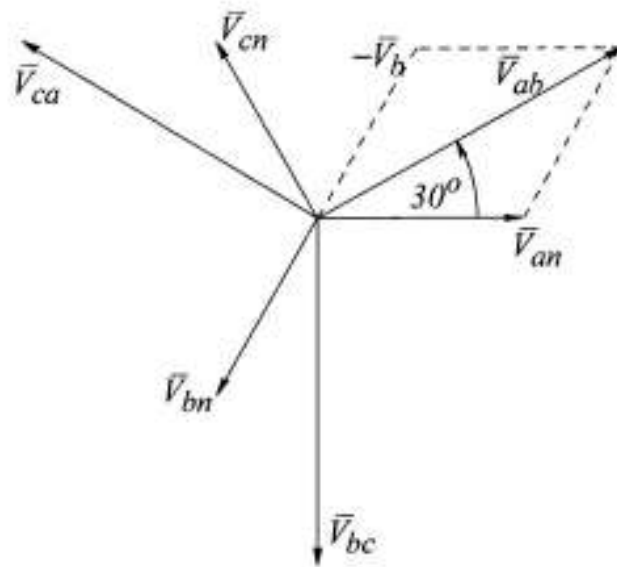
# Balanced Three-Phase Circuits Including Mutual Coupling



- Example: in a 3-phase synchronous generator the flux lines produced by a phase current link not only that phase winding but the other two phase windings as well

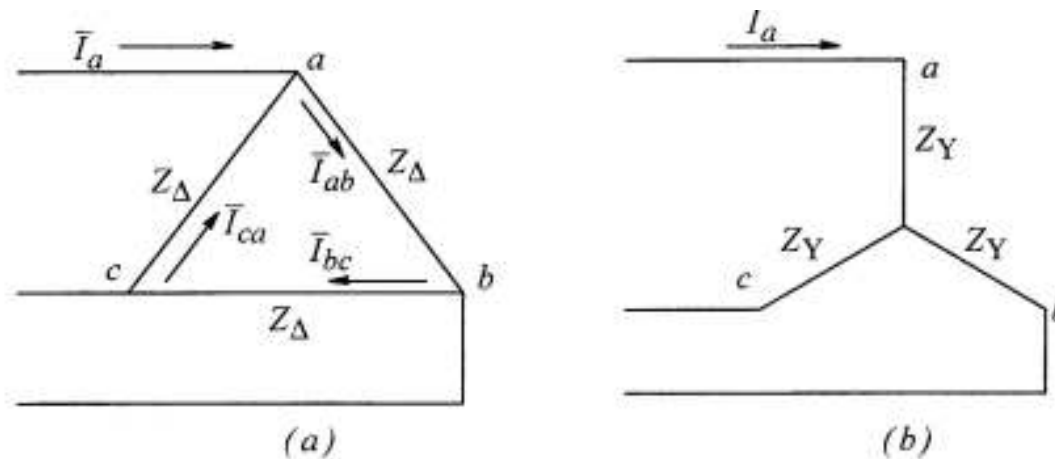


# Line-to-Line Voltages



Line-to-line voltages in a three-phase Y connected circuit – phasor diagram

# Delta Connected Loads



Delta-to-wye transformation of a three-phase load

- Example: the windings in 3-phase ac motor drives, or transformers
- Under balanced conditions determine  $Z_Y = Z_\Delta/3$ , and then apply a single phase equivalent analysis

# Powers in Three-Phase Circuits

The total average (real) and reactive powers in a 3-phase circuit are

$$P_{3-phase} = 3 \times P_{per-phase} = 3VI \cos \phi$$

$$Q_{3-phase} = 3 \times Q_{per-phase} = 3VI \sin \phi$$

where  $V = V_{ph}$  and  $I = I_{ph}$  represent per-phase *rms* values

The total apparent power in a 3-phase circuit is

$$|S|_{3-phase} = 3 \times |S|_{per-phase} = 3VI$$

# Powers in Three-Phase Circuits

The power factor in a three-phase circuit is the same as the per-phase power factor

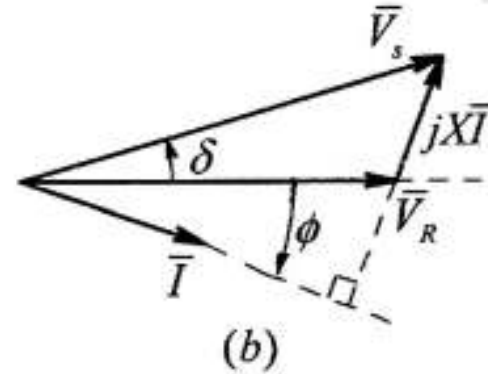
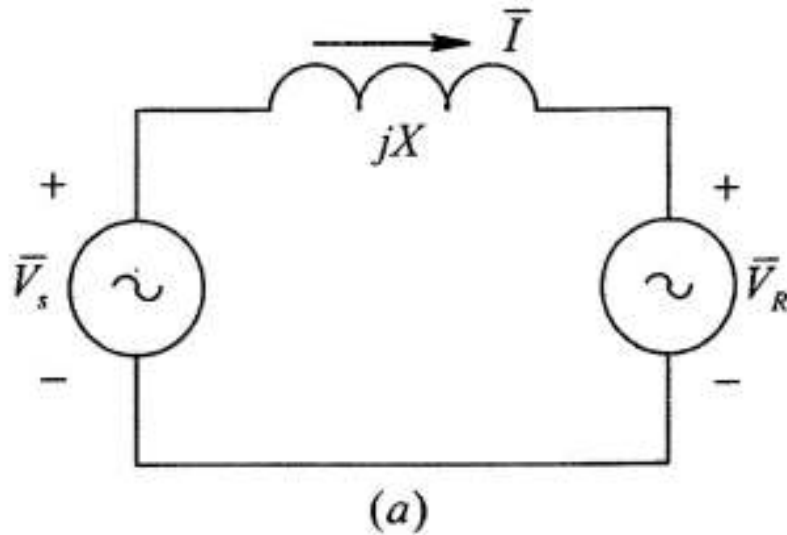
$$\text{pf} = P_{3\text{-phase}} / |S|_{3\text{-phase}} = 3VI\cos\varphi / (3VI) = \cos\varphi$$

Specific to 3-phase circuits is the total (combined) instantaneous power in steady-state which is constant (non-pulsating), and equal to its average value

$$\begin{aligned} p_{3\text{-phase}}(t) &= p_a(t) + p_b(t) + p_c(t) \\ &= 3VI\cos\varphi = P_{3\text{-phase}} \end{aligned}$$

➤ *The non-pulsating  $p_{3\text{-phase}}$  results in non-pulsating torque in 3-phase motors and generators*

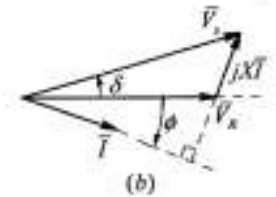
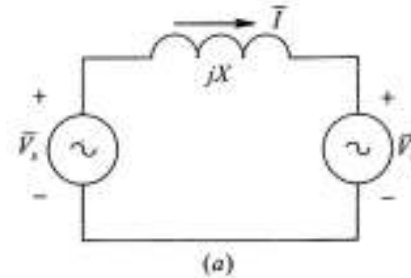
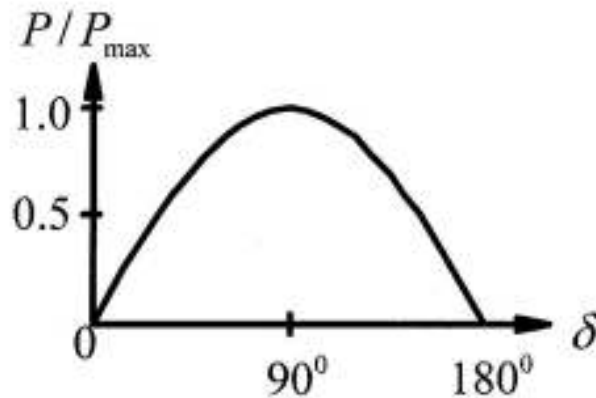
# Power Flow in AC Systems



Power transfer between two ac systems  
in a simplified representation

$$\text{➤ } S_R = P_R + j Q_R = V_R I^*$$

# Power-Angle Diagram



Power as a function of the phase angle (of the  $V_s$  voltage)

➤  $P_R = (V_S V_R \sin \delta) / X$ ;  $Q_R = (V_S V_R \cos \delta) / X - V_R V_R / X$

➤ At  $\sin \delta = 0$ ,  $\delta = 0$ , and then  $Q_R = V_R / X (V_S - V_R)$

# Base Values and Per Unit Quantities

$$R_{base}, X_{base}, Z_{base} = \frac{V_{base}}{I_{base}} \quad (\text{in } \Omega)$$

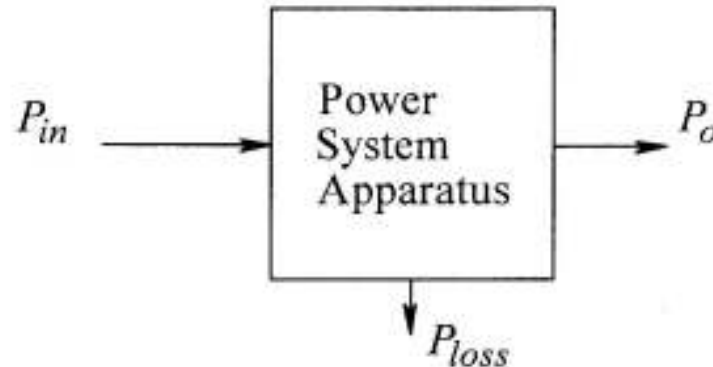
$$G_{base}, B_{base}, Y_{base} = \frac{I_{base}}{V_{base}} \quad (\text{in } \mathcal{S})$$

$$P_{base}, Q_{base}, (VA)_{base} = V_{base} I_{base} \quad (\text{in Watt, VAR, or VA})$$

In terms of these base quantities, the per-unit quantities can be specified as

$$\text{Per-Unit Value} = \frac{\text{actual value}}{\text{base value}}$$

# Energy Efficiency of Power System Apparatus



➤ Energy efficiency :  $\eta = P_o / P_{in} = P_o / (P_o + P_{loss})$

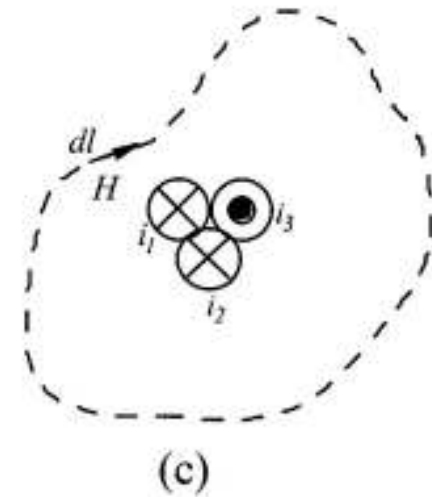
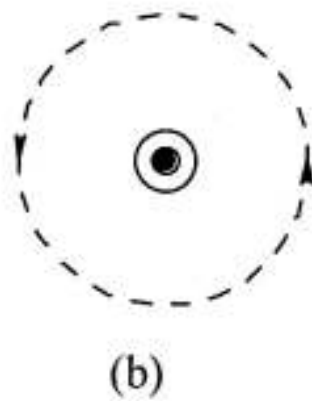
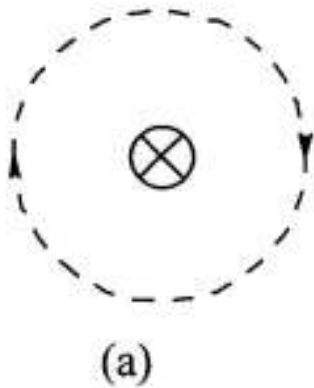


**Thank You!**

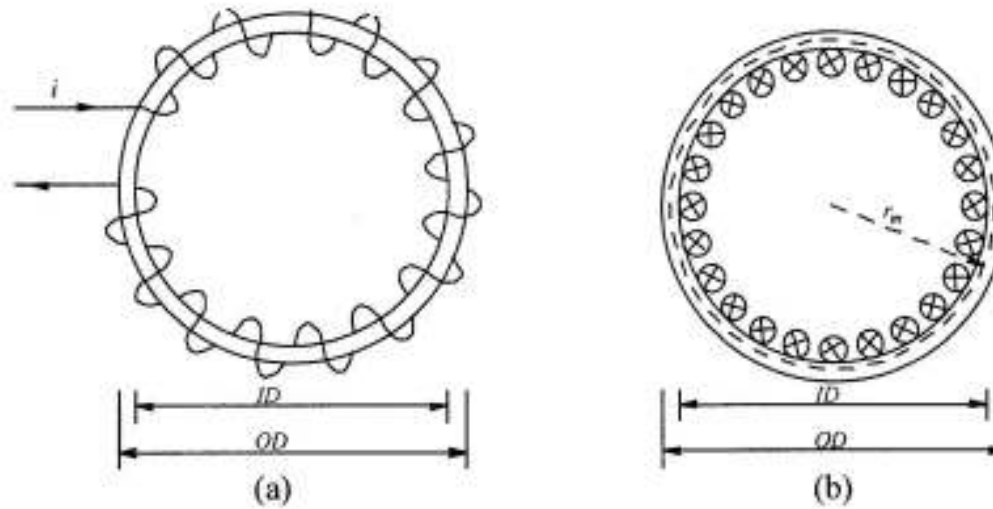
# Discussion

# Electromagnetic Concepts

## Ampere's law

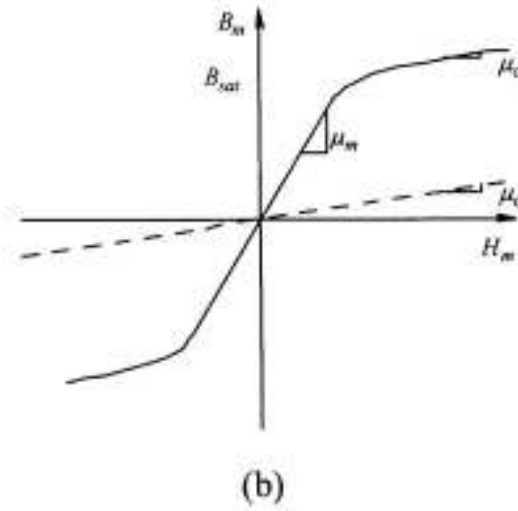
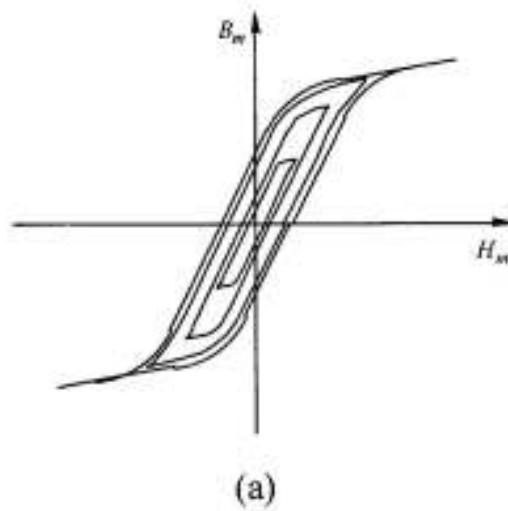


# Example of a Toroid



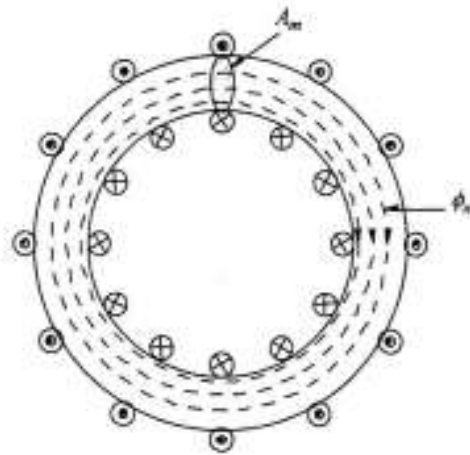
# Ferromagnetic Materials

## B-H Curves



B-H characteristic of ferromagnetic materials

# Magnetic Flux $\phi_m$ and Flux-Density $B$



Toroid with flux  $\phi_m$

# Flux Linkage and Magnetizing Inductance

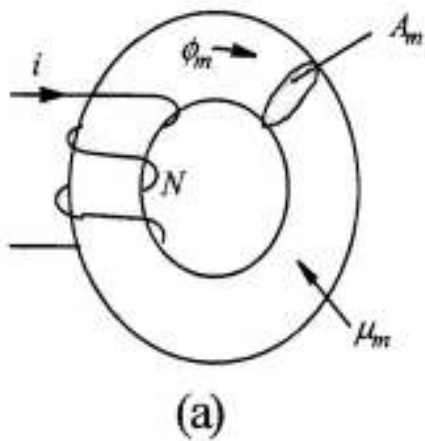
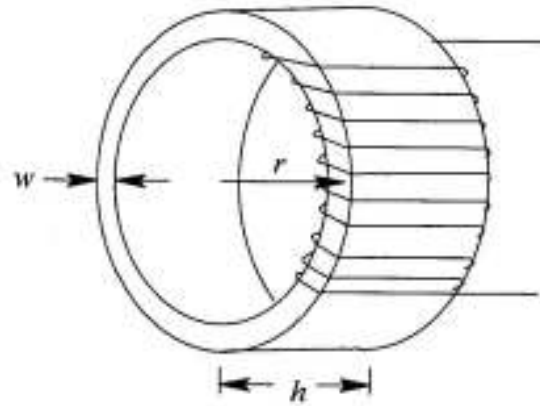


Diagram (b) shows the relationship between current  $i$ , magnetic field intensity  $H_m$ , magnetic flux density  $B_m$ , magnetic flux  $\phi_m$ , and flux linkage  $\lambda_m$ . The relationship is summarized by the equation:

$$L_m = \frac{N^2}{\frac{\ell_m}{\mu_m A_m}}$$

# Example of a Toroid

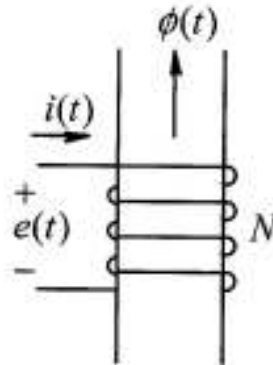


Rectangular toroid



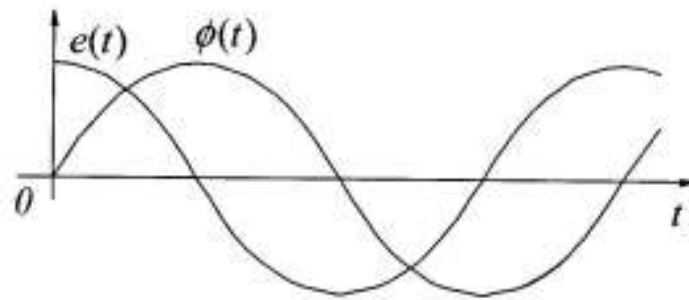
# Faraday's Law

## Induced Voltage in a Coil

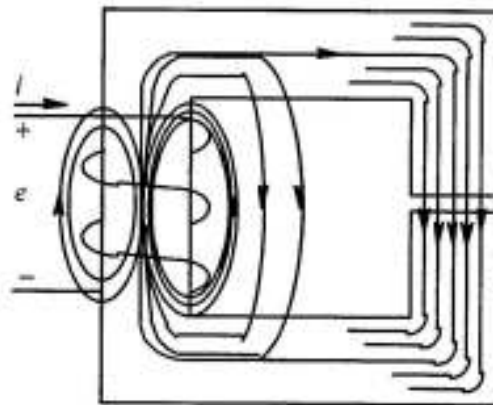


Voltage polarity and direction of flux and current

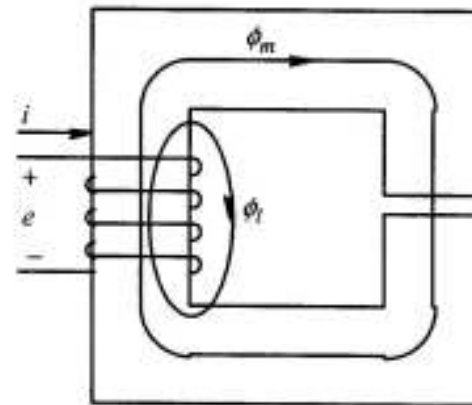
# Plot of Time-Varying Flux and Induced Voltage



# Leakage Flux $\phi_l$



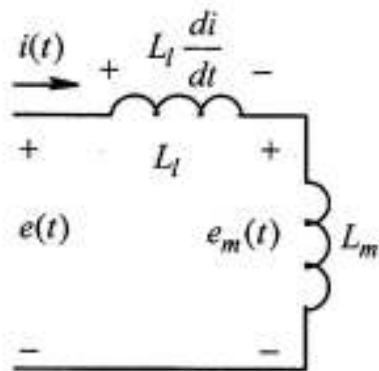
(a)



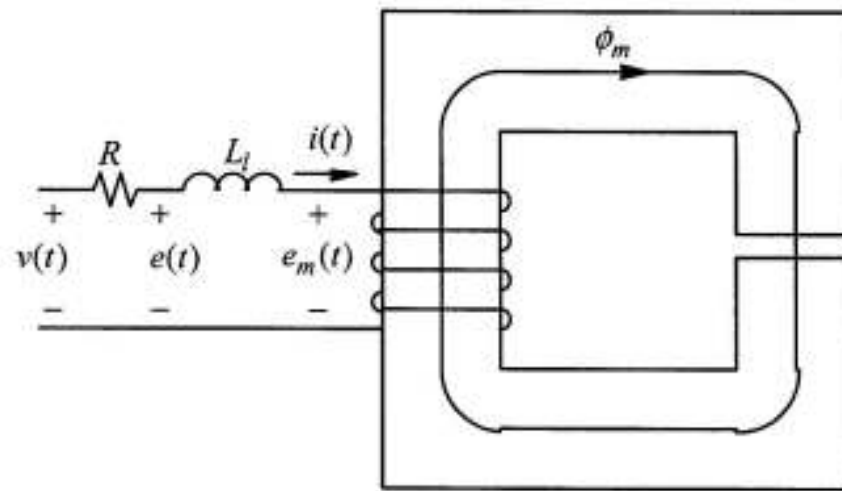
(b)

Magnetic field including leakage flux

# Use of Leakage Inductance $L_l$ to Represent the Leakage Flux $\phi_l$



(a)



(b)

Analysis including the leakage flux

New Discussion